ECED2200 – Digital Circuits

Boolean Algebra

General Notes

- See updates to these slides: www.newae.com/teaching
- These slides licensed under 'Creative Commons Attribution-ShareAlike 3.0 Unported License'
- These slides are not the complete course – they are extended in-class
- You will find the following references useful, see www.newae.com/teaching for more information/links:
  - The book "Bebop to the Boolean Boogie" which is available to Dalhousie Students
  - Course notes (covers almost everything we will discuss in class)
  - Various websites such as e.g.: www.play-hookey.com
  - The book "Contemporary Logic Design", which was used in previous iterations of the class and you may have already

BOOLEAN ALGEBRA

No.	Identity	Comments
1	$A+0=A$	Operations with 0 and 1
2	$A+1=1$	Operations with 0 and 1
3	$A+A=A$	Idempotent
4	$A+\overline{A}=1$	Complements
5	$A\bullet 0=0$	Operations with 0 and 1
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12	$A+(B+C)=(A+B)+C=A+B+C$	Associative
13	$A\bullet(B\bullet C)=(A\bullet B)\bullet C=A\bullet B\bullet C$	Associative
14	$A\bullet(B+C)=(A\bullet B)+(A\bullet C)$	Distributive
15	$A+(B\bullet C)=(A+B)\bullet(A+C)$	Distributive
16	$A+(A\bullet B)=A$	Absorption
17	$A\bullet(A+B)=A$	Absorption
18	$(A\bullet B)+(\overline{A}\bullet C)+(B\bullet C)=(A\bullet B)+(\overline{A}\bullet C)$	Consensus
19	$\overline{A+B+C+\ldots}=\overline{A}\bullet\overline{B}\bullet\overline{C}\ldots$	De Morgan
20	$\overline{A\bullet B\bullet C\bullet\ldots}=\overline{A}+\overline{B}+\overline{C}\ldots$	De Morgan
21	$(A+\overline{B})\bullet B=A\bullet B$	Simplification
22	$(A\bullet\overline{B})+B=A+B$	Simplification

Operations with 0 and 1

$A+0=A$

$A+1=1$

$A \bullet 0=0$

$A \bullet 1=A$

Idempotent Rules

$$A + A = A$$

$$A \bullet A = A$$

Complementary Rules

$$A+\overline{A}=1$$

$$A \bullet \overline{A}=0$$

Involution

$$\overline{\overline{A}} = A$$

Commutative

$$A+B=B+A$$

$$A \bullet B=B \bullet A$$

Associative Rules

$$A+(B+C)=(A+B)+C=A+B+C$$

$$A\bullet(B\bullet C)=(A\bullet B)\bullet C=A\bullet B\bullet C$$

Distributive

$$A \bullet (B+C) = (A \bullet B) + (A \bullet C)$$

$$A + (B \bullet C) = (A+B) \bullet (A+C)$$

Order of Operations

$$Y = A + \overline{B} \bullet C$$

Absorption

$$A + (A \bullet B) = A$$
$$A \bullet (A + B) = A$$

Consensus

$$(A \bullet B)+(\overline{A} \bullet C)+(B \bullet C)=(A \bullet B)+(\overline{A} \bullet C)$$

DeMorgan

$$\overline{A+B+C+\ldots}=\overline{A}\bullet\overline{B}\bullet\overline{C}\ldots$$

$$\overline{A\bullet B\bullet C\bullet\ldots}=\overline{A}+\overline{B}+\overline{C}\ldots$$

Simplification

$$(A+\overline{B})\bullet B=A\bullet B$$

$$(A\bullet\overline{B})+B=A+B$$

$$A+(A\bullet B)=A$$

$$A\bullet(A+B)=A$$

Proof by Perfect Induction

$$(A+\overline{B})\bullet B=A\bullet B$$

A	B	$+\overline{B})$	$(A+\overline{B})$	Y
0	0			
0	1			
1	0			
1	1			

A	B	Y
0	0	
0	1	
1	0	
1	1	

Deriving Identities

$A+(A \bullet B)=A$

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Deriving Identities

$A+(A\bullet B)$

$= (A+A)\bullet(A+B)$ [15]

$=A\bullet(A+B)$ [3]

$=A\bullet A+A\bullet B$ [14]

$=A\bullet 1+A\bullet B$ [6]

$=A\bullet(1+B)$ [14]

$=A\bullet 1$ [2]

$=A$ [6]

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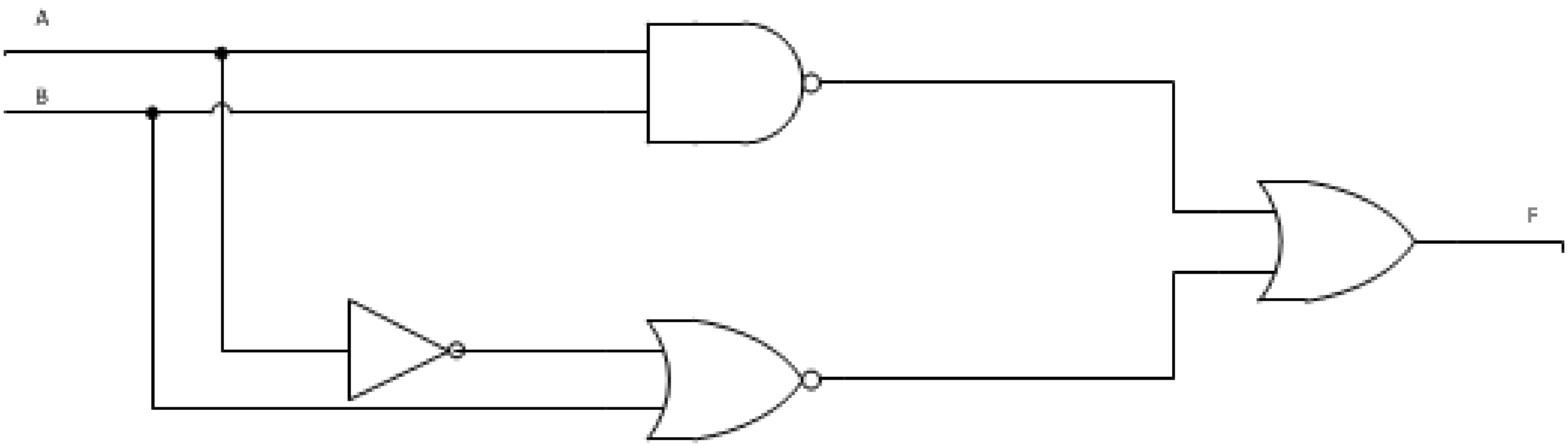
Simplifying Circuits



Simplifying Circuits

$$f = A \bullet B + \overline{\overline{A} + B}$$ Initial

$$= \left(\overline{A} + \overline{B}\right) + \left(A \bullet \overline{B}\right)$$ Using 19 & 20 – DeMorgans

$$= \overline{A} + 1 \bullet \overline{B} + \left(A \bullet \overline{B}\right)$$ Using 6 & dropping brackets

$$= \overline{A} + (1 + A) \bullet \overline{B}$$ Using 14 – note we go from distributed form to compact form, where B term is common

$$= A + 1 \bullet \overline{B}$$ Using 2

$$= \overline{A} + \overline{B}$$ Using 6

$$= \overline{A \bullet B}$$ Using 20 - DeMorgans

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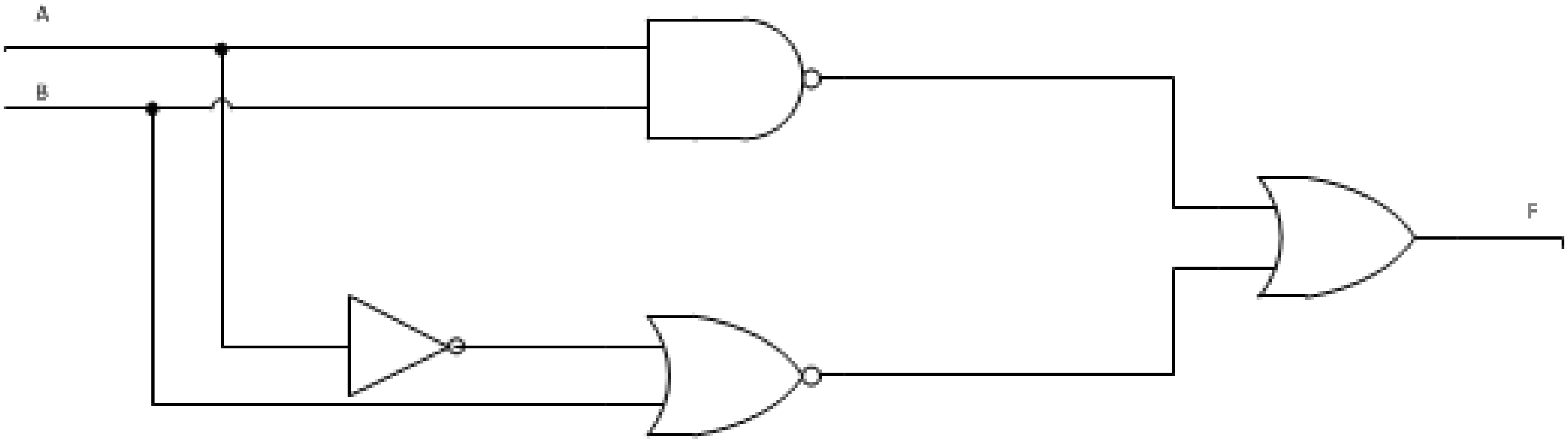
Simplifying Circuits



Is really just a NAND gate! Thanks Boolean Algebra!

Additional Examples #1

$A \bullet (A+B) = A$

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22	$(A \bullet \overline{B})+B=A+B$	Simplification

Additional Examples

$$A \bullet (A+B)=A$$
$$=A \bullet A+A \bullet B$$
$$=A \bullet 1+A \bullet B$$
$$=A \bullet (1+B)$$
$$=A$$

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Additional Examples #2

$$f = \overline{A \bullet B + \overline{A} \bullet \overline{B}} + \overline{A} \bullet B$$

No.	Identity	Comments
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3	$A+A=A$	Idempotent
4	$A+\overline{A}=1$	Complements
5	$A \bullet 0=0$	Operations with 0 and 1
6	$A \bullet 1=A$	Operations with 0 and 1
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22	$(A \bullet \overline{B})+B=A+B$	Simplification

Additional Examples

$$f=\overline{A\bullet B+\overline{A}\bullet\overline{B}}+\overline{A}\bullet B$$

$$=\left(\overline{A\bullet B}\right)\bullet\left(\overline{\overline{A}\bullet\overline{B}}\right)+\overline{A}\bullet B$$

$$=\left(\overline{A}+\overline{B}\right)\bullet\left(A+B\right)+\overline{A}\bullet B$$

$$=\overline{A}\bullet A+\overline{A}\bullet B+\overline{B}\bullet A+\overline{B}\bullet B+\overline{A}\bullet B$$

$$=0+\overline{A}\bullet B+\overline{B}\bullet A+0+\overline{A}\bullet B$$

$$=\overline{A}\bullet B+\overline{A}\bullet B+\overline{B}\bullet A$$

$$=\overline{A}\bullet B+\overline{B}\bullet A$$

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Additional Examples #2 NAND Only

$$f=\overline{A\bullet B+\overline{A}\bullet\overline{B}}+\overline{A}\bullet\overline{B}$$

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Additional Examples #2 NAND Only

$$f = \overline{A\bullet B + \overline{A}\bullet\overline{B}} + \overline{A}\bullet\overline{B}$$

$$= \overline{A}\bullet B + \overline{B}\bullet A$$

$$= \overline{\overline{\overline{A}\bullet B + \overline{B}\bullet A}}$$

$$= \overline{\overline{\overline{A}\bullet B}\bullet\overline{\overline{B}\bullet A}}$$

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Section Summary

- Bebop to the Boolean Boogie: Chapter 9
- Contemporary Logic Design: Chapter 2
- ECED Notes: “Boolean Algebra” (Page 48)

Checking your simplifications:

http://joshtam.net/world/bee-calc.html

CANONICAL FORMS

Designing Logic Circuits

A	B	C	Output (f)
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

Minterms & Maxterms

A	B	C	Minterms	Maxterms
0	0	0	$m_0 = \overline{A} \bullet \overline{B} \bullet \overline{C}$	$M_0 = A + B + C$
0	0	1	$m_1 = \overline{A} \bullet \overline{B} \bullet C$	$M_1 = A + B + \overline{C}$
0	1	0	$m_2 = \overline{A} \bullet B \bullet \overline{C}$	$M_2 = A + \overline{B} + C$
0	1	1	$m_3 = \overline{A} \bullet B \bullet C$	$M_3 = A + \overline{B} + \overline{C}$
1	0	0	$m_4 = A \bullet \overline{B} \bullet \overline{C}$	$M_4 = \overline{A} + B + C$
1	0	1	$m_5 = A \bullet \overline{B} \bullet C$	$M_5 = \overline{A} + B + \overline{C}$
1	1	0	$m_6 = A \bullet B \bullet \overline{C}$	$M_6 = \overline{A} + \overline{B} + C$
1	1	1	$m_7 = A \bullet B \bullet C$	$M_7 = \overline{A} + \overline{B} + \overline{C}$

Minterm/Maxterm

Minterm = *product* (AND) of input variable

Maxterm = *sum* (OR) of inverted input variable

Sum of Products

A	B	C	Minterms	f
0	0	0		0
0	0	1		0
0	1	0		0
0	1	1	$m_3 = \overline{\mathrm{A}} \bullet \mathrm{B} \bullet \mathrm{C}$	1
1	0	0	$m_4 = \mathrm{A} \bullet \overline{\mathrm{B}} \bullet \overline{\mathrm{C}}$	1
1	0	1	$m_5 = \mathrm{A} \bullet \overline{\mathrm{B}} \bullet \mathrm{C}$	1
1	1	0	$m_6 = \mathrm{A} \bullet \mathrm{B} \bullet \overline{\mathrm{C}}$	1
1	1	1	$m_7 = \mathrm{A} \bullet \mathrm{B} \bullet \mathrm{C}$	1

$$f = m_3 + m_4 + m_5 + m_6 + m_7$$

Sum of Products

$$f(A,B,C)=\sum m_i(3,4,5,6,7)$$

Sum of Products

$$f = \overline{A} \bullet B \bullet C + A \bullet \overline{B} \bullet \overline{C} + A \bullet \overline{B} \bullet C + A \bullet B \bullet \overline{C} + A \bullet B \bullet C$$

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18	$(A\bullet B)+(\overline{A}\bullet C)+(B\bullet C)=(A\bullet B)+(\overline{A}\bullet C)$	Consensus
19	$\overline{A+B+C+\ldots}=\overline{A}\bullet\overline{B}\bullet\overline{C}\ldots$	De Morgan
20	$\overline{A\bullet B\bullet C\bullet\ldots}=\overline{A}+\overline{B}+\overline{C}\ldots$	De Morgan
21	$(A+\overline{B})\bullet B=A\bullet B$	Simplification
22	$(A\bullet\overline{B})+B=A+B$	Simplification

Sum of Products

$$f = \overline{A}\bullet B\bullet C + A\bullet\overline{B}\bullet\overline{C} + A\bullet\overline{B}\bullet C + A\bullet B\bullet\overline{C} + A\bullet B\bullet C$$

$$= \overline{A}\bullet B\bullet C + A\bullet\overline{B}\bullet(C+\overline{C}) + A\bullet B\bullet(C+\overline{C})$$

$$= \overline{A}\bullet B\bullet C + A\bullet\overline{B} + A\bullet B$$

$$= \overline{A}\bullet B\bullet C + A\bullet(\overline{B} + B)$$

$$= \overline{A}\bullet B\bullet C + A$$

$$= B\bullet C + A$$

Using No.22, where:

B*C = A’

A =B’

No.	Identity	Comments
1	$A+0=A$	Operations with 0 and 1
2	$A+1=1$	Operations with 0 and 1
3	$A+A=A$	Idempotent
4	$A+\overline{A}=1$	Complements
5	$A\bullet 0=0$	Operations with 0 and 1
6	$A\bullet 1=A$	Operations with 0 and 1
7	$A\bullet A=A$	Idempotent
8	$A\bullet\overline{A}=0$	Complements
9	$\overline{\overline{A}}=A$	
10	$A+B=B+A$	Commutative
11	$A\bullet B=B\bullet A$	Commutative
12	$A+(B+C)=(A+B)+C=A+B+C$	Associative
13	$A\bullet(B\bullet C)=(A\bullet B)\bullet C=A\bullet B\bullet C$	Associative
14	$A\bullet(B+C)=(A\bullet B)+(A\bullet C)$	Distributive
15	$A+(B\bullet C)=(A+B)\bullet(A+C)$	Distributive
16	$A+(A\bullet B)=A$	Absorption
17	$A\bullet(A+B)=A$	Absorption
18	$(A\bullet B)+(\overline{A}\bullet C)+(B\bullet C)=(A\bullet B)+(\overline{A}\bullet C)$	Consensus
19	$\overline{A+B+C+\ldots}=\overline{A}\bullet\overline{B}\bullet\overline{C}\ldots$	De Morgan
20	$\overline{A\bullet B\bullet C\bullet\ldots}=\overline{A}+\overline{B}+\overline{C}\ldots$	De Morgan
21	$(A+\overline{B})\bullet B=A\bullet B$	Simplification
22	$(A\bullet\overline{B})+B=A+B$	Simplification

Product of Sums

A	B	C	Maxterms	f
0	0	0	$M_0 = A+B+C$	0
0	0	1	$M_1 = A+B+\overline{C}$	0
0	1	0	$M_2 = A+\overline{B}+C$	0
0	1	1		1
1	0	0		1
1	0	1		1
1	1	0		1
1	1	1		1

$$f = M_0 \bullet M_1 \bullet M_2$$

Product of Sums

$$f(A,B,C) = \Pi M_i(0,1,2)$$

Product of Sums

$$f = (A+B+C)\bullet(A+B+\overline{C})\bullet(A+\overline{B}+C)$$

No.	Identity	Comments
1	$A+0=A$	Operations with 0 and 1
2	$A+1=1$	Operations with 0 and 1
3	$A+A=A$	Idempotent
4	$A+\overline{A}=1$	Complements
5	$A\bullet 0=0$	Operations with 0 and 1
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7	$A\bullet A=A$	Idempotent
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9	$\overline{\overline{A}}=A$	
10	$A+B=B+A$	Commutative
11	$A\bullet B=B\bullet A$	Commutative
12	$A+(B+C)=(A+B)+C=A+B+C$	Associative
13	$A\bullet(B\bullet C)=(A\bullet B)\bullet C=A\bullet B\bullet C$	Associative
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18	$(A\bullet B)+(\overline{A}\bullet C)+(B\bullet C)=(A\bullet B)+(\overline{A}\bullet C)$	Consensus
19	$\overline{A+B+C+\ldots}=\overline{A}\bullet\overline{B}\bullet\overline{C}\ldots$	De Morgan
20	$\overline{A\bullet B\bullet C\bullet\ldots}=\overline{A}+\overline{B}+\overline{C}\ldots$	De Morgan
21	$(A+\overline{B})\bullet B=A\bullet B$	Simplification
22	$(A\bullet\overline{B})+B=A+B$	Simplification

Product of Sums

$$f=(A+B+C)\bullet(A+B+\overline{C})\bullet(A+\overline{B}+C)$$

$$=\left[(A+B)+(C\bullet\overline{C})\right]\bullet(A+\overline{B}+C)$$

$$=(A+B)\bullet(A+\overline{B}+C)$$

$$=A+\left(B\bullet(\overline{B}+C)\right)$$

$$=A+(B\bullet\overline{B}+B\bullet C)$$

$$=A+B\bullet C$$

No.	Identity	Comments
1	$A+0=A$	Operations with 0 and 1
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18	$(A\bullet B)+(\overline{A}\bullet C)+(B\bullet C)=(A\bullet B)+(\overline{A}\bullet C)$	Consensus
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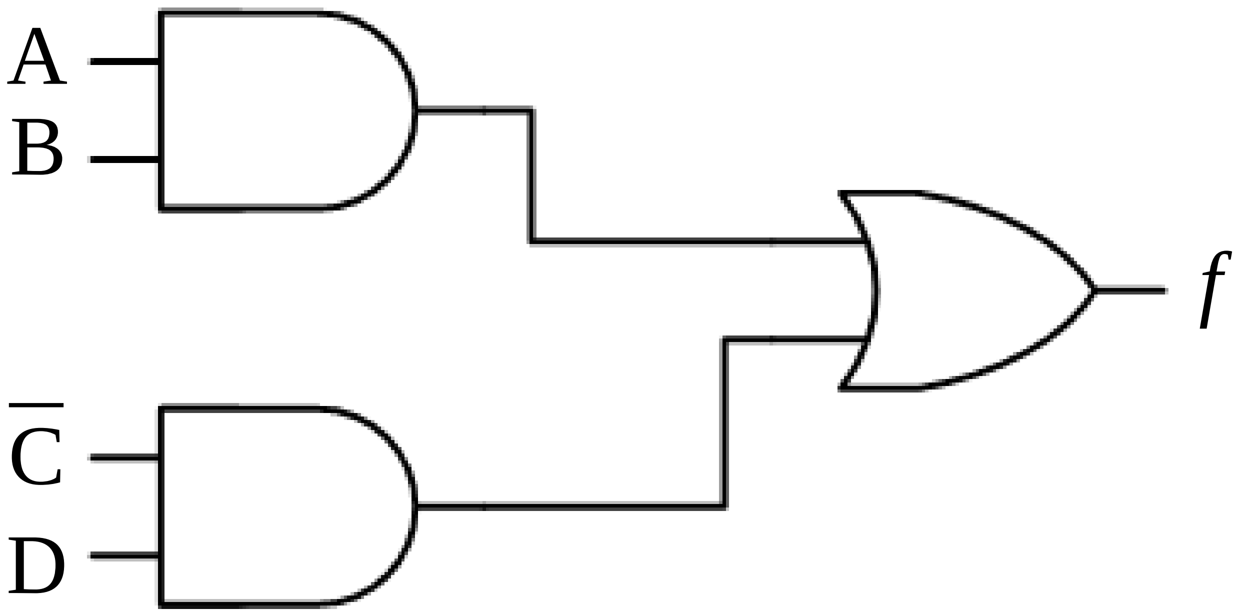
20	$\overline{A\bullet B\bullet C\bullet\ldots}=\overline{A}+\overline{B}+\overline{C}\ldots$	De Morgan
21	$(A+\overline{B})\bullet B=A\bullet B$	Simplification
22	$(A\bullet\overline{B})+B=A+B$	Simplification

Two-Level Combinational Logic



$$f = (A \cdot B) + (\overline{C} \cdot D)$$

Section Summary

- Bebop to the Boolean Boogie: Chapter 9
- Contemporary Logic Design: Chapter 2
- ECED Notes: “Two Level Canonical Forms” (Page 69)